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Torsion and Gauge-Invariant Massive Electrodynamics

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We study electrodynamics in Einstein–Cartan space-time, that is, in space-time with torsion, and show an analogy with the Chern–Simons gauge-invariant massive electrodynamics. In our case, however, there is no arbitrary parameter, the torsion Q playing the role of the Chern–Simons parameter κ. This leads to bounds on the photon mass, charge, and torsion coupling.

1. INTRODUCTION

Whenever a photon mass is invoked in electrodynamics, as has been done by several authors at different times (Einstein, 1917; Bass and Schrödinger, 1955; Garcia de Andrade, 1990a; Chow, 1981; Barnes and Sergle, 1975), gauge invariance is necessarily broken, as is well known. So whenever limits are put on a photon mass in different contexts (Chow, 1981; Barnes and Sergle, 1975), it is understood that gauge invariance and conformal invariance of Maxwell's equations are broken. As regards the conformal invariance of Maxwell's equations, once we introduce a curved space-time the propagation of a photon is no longer conformally invariant (i.e., we have the bending of light). Only in the special case of a conformally flat space are Maxwell's equations conformally invariant. In general in a curved space-time, conformal invariance of Maxwell's equations is broken, so that the curvature term can be interpreted as a mass term. For instance, general covariance would modify the wave equation as

$$\Box A^{\mu} + R^{\mu}{}_{\nu} A^{\nu} = 0 \tag{1}$$

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However, it must be noted that this is not unique. One could write down as well $\Box A^\mu + R_{\alpha\nu} R^{\alpha\nu} A^\nu = 0$ or even $\Box A^\mu + R_{\gamma\nu\alpha\beta} R^{\gamma\nu\alpha\beta} A^\mu = 0$, etc. All these expressions would be generally covariant. The photon mass would be related or constrained by the overall curvature of space as $m_\gamma^2 \sim R$.

The above considerations were for a curved space with a symmetrical connection. For a curved space with a nonsymmetrical connection it is also well known that minimal coupling does not preserve gauge invariance (Hehl *et al.*, 1976; Hayashi and Sasaki, 1978). Suggested solutions involve invoking additional constraints on torsion or introduce other coupling prescriptions (Novello, 1976).

Classically one can of course retain gauge invariance by assuming that photons do not couple to the background torsion field. This complete decoupling could of course be valid only classically. The quantum description of the electromagnetic field would imply that a photon can for a small fraction of time disintegrate into a virtual fermion pair and since massive fermions can minimally couple to torsion, this pair can interact with torsionic background, inducing indirectly a photon–torsion coupling which can preserve the gauge invariance of the theory.

The Maxwell equations modified in this manner in the presence of torsion can be written as (de Sabbata and Gasperini, 1980a, b; 1981a–c)

$$\begin{aligned} \partial_k F^{ik} &= 4\pi J^i + (2\alpha/3\pi)\eta^{iklj} F_{kl} Q_j \\ \partial_{[i} F_{jk]} &= 0 \end{aligned} \tag{2}$$

($\alpha = e^2/\hbar c$). These equations are compatible with current conservation, $\partial_i J^i = 0$ (in the case of constant torsion, i.e., $\partial_i Q_j = 0$), but are *fully gauge invariant.*

In this modification of Maxwell's equation in the presence of background torsion we effectively have an "extra" contribution to the source term on the r.h.s. of the first of equations (2), something like a correction to the current density J^i. Also we note that the parameter Q_j, which has the dimensionality of *inverse length*, controls the torsion modification. Q can be defined through the spin density σ of the background matter as (de Sabbata and Gasperini, 1980c)

$$Q = 4\pi G\sigma/c^3 \tag{3}$$

Q as defined in equation (3) has the dimensions of inverse length. This additional term in equation (2) can thus be pictured as providing a wavelength cutoff at large distances greater than Q^{-1} for the gauge-invariant electric and magnetic fields. In the static case, we have a correction to the charge density.

We shall now point out some analogies between this additional gauge-invariant term induced in the electrodynamic equations by torsion and the Chern–Simons modification of electrodynamics.

2. GAUGE-INVARIANT MASSIVE ELECTRODYNAMICS

The gauge-invariant modification of electrodynamics introduced by a torsion background as discussed above has a parallel with the Chern–Simons modification of electrodynamics, which is currently receiving much attention (Jackiw and So-Young Pi, 1990). With the Chern–Simons term we have the modified Maxwell equation given as

$$\partial_\mu F^{\mu\nu} + (\kappa/2)\varepsilon^{\nu\alpha\beta}F_{\alpha\beta} = 4\pi J^\nu \tag{4}$$

where the parameter κ has inverse length dimensionality and controls the additional Chern–Simons term. This additional term gives rise again to a massive yet *gauge-invariant* electrodynamics.

This is the usual interpretation of the Chern–Simons term, which is a coupling between field and potential $A_\mu F^{\mu\nu}$, analogous to the coupling between torsion and field, i.e., here Q plays the role of the Chern–Simons parameter κ.

The Lagrangian density for Chern–Simons electrodynamics involves coupling between field $F^{\mu\nu}$ and potential A_μ thus

$$\mathscr{L} = -(1/4)F_{\mu\nu}F^{\mu\nu} + (\kappa/4)\varepsilon^{\mu\nu\lambda}F_{\mu\nu}A_\lambda \tag{5}$$

The resulting field equations given by equation (4) are invariant under

$$A_\mu \Rightarrow A_\mu + (1/c)\partial_\mu \Lambda \tag{6}$$

The torsion-modified part of the electrodynamic action is given by (de Sabbata and Gasperini, 1981a)

$$L = (\alpha/16\pi)\int d^4x\,(-g)^{1/2}\varepsilon^{ijk}A_i F_{jk} Q \tag{7}$$

which is also invariant under the gauge transformation given by equation (6).

Q is defined in terms of the background spin density through equation (3) and being an inverse length is formally similar to the inverse-length Chern–Simons parameter κ (de Sabbata and Gasperini, 1981d). Notice also the field-potential coupling in equation (7). The fact that equations (5) and (7) correspond to a fully gauge-invariant theory with massive photons can be seen by rewriting, for instance, equation (4) using the dual fields given by

$$^*F_\nu = (1/2)\varepsilon_{\nu\alpha\beta}F^{\alpha\beta} \tag{8}$$

Thus equation (4) becomes

$$(g^{\mu\nu} + (1/\kappa)\varepsilon^{\mu\nu\alpha}\partial_\alpha)\ ^*F_\nu = 0 \tag{9}$$

which we can write also in the form

$$(\Box + \kappa^2)\ ^*F_\beta = 0 \tag{10}$$

where the photon mass is related to κ as

$$(m_\gamma c/\hbar)^2 = \kappa^2 \tag{11}$$

In the torsion case, m_γ^2 is related to Q^2, with Q given by equation (3).

So essentially what we have is a case of massive yet gauge-invariant electrodynamics, "massive" in the sense that we have a length parameter Q^{-1} entering as a cutoff (determined by torsion) which can be considered as equivalent to a mass term if one associates it with the corresponding Compton wavelength, thus $m_\gamma \propto Q$. For vanishing torsion $Q=0$, i.e., $Q^{-1}=\infty$, we recover the usual Maxwell electrodynamics with infinite field range and zero photon mass. We thus have the important consequence that we have no infrared divergence in electrodynamics, as we have a wavelength cutoff at Q^{-1}. For instance, on a cosmic scale Q arising from background torsion due to the spin density of matter in the universe has a value of $Q \approx R_{\mathrm{H}}^{-1}/\alpha$, giving the effective cutoff wavelength on a cosmic scale as $\approx R_{\mathrm{H}}\alpha \approx 10^{26}$ cm, R_{H} being the Hubble radius. This corresponds to an effective photon mass (from $\hbar/m_\gamma c = R_{\mathrm{H}}\alpha$) of $m_\gamma \approx 10^{-62}$ g. This mass for the photon is only an effective mass which is induced by the background cosmological torsion. It is not something intrinsic to the photon mass (like the W-boson mass, for example). This is only an effective mass induced by its propagation through the background spin density, so that gauge invariance is preserved. Moreover, the photon is not a gravitationally bound system (either by Newtonian or strong gravity). In this sense it is remarkable to talk of an effective photon mass on a cosmic scale.

The additional term in equation (4) would give rise to a modified Gauss law as

$$\nabla E - \kappa B = e\rho(r, t) \tag{12}$$

where $\rho(r, t)$ is the charge number density. This has the consequence that any field configuration with charge $q = e\int dr\, \rho(r, t)$ also carries a magnetic flux $\Phi = \int dr\, B(r, t)$. As remarked earlier, the additional term, which is still gauge invariant, can be regarded as contributing an "extra" charge which would give rise to a background magnetic field (in turn related to Q). For a discussion of the torsion-induced primordial magnetic field see de Sabbata and Sivaram (1988).

The main effect of torsion modification consists of introducing a *gauge-invariant* infrared cutoff Q^{-1} playing the role of a finite range analogous to the usual interpretation of Chern–Simons electrodynamics as a massive gauge-invariant theory with the range entering through κ^{-1}. We have also the corresponding divergence of the axial current as

$$\partial^{\mu} j_{\mu 5} = e^2 \varepsilon_{\mu\nu\lambda\sigma} F^{\mu\nu} F^{\lambda\sigma} \tag{13}$$

For the torsion case, where the spin–torsion coupling arises from the axial-vector part of the torsion tensor (de Sabbata and Gasperini, 1980a; de Sabbata and Sivaram, 1990a) written as $Q = \partial_{\mu}\phi$, we have analogously (here we consider propagating torsion, since we are considering field theory)

$$\partial Q \propto \Box\phi = (G\alpha/c^5)\varepsilon^{ijkl} F_{ij} F_{kl} \tag{14}$$

The approach can be easily extended to the non-Abelian case, the corresponding gauge groups being $SU(2)$ isospin for the Chern–Simons case and $SL(2, C)$ for the torsion case (de Sabbata and Sivaram, 1991). In this respect the Chern–Simons approach is a particular case of torsion development.

It is also interesting to note that in the Chern–Simons case, κ^{-1} (i.e., the length parameter) is quantized in units of g^2/mc^2, where g is the gauge coupling, i.e.,

$$\kappa^{-1} = (g^2/mc^2)\, n \tag{15}$$

where n is an integer. Similarly in the torsion case, if we associate torsion with defects in space-time topology (de Sabbata and Sivaram, 1991), Q^{-1} is quantized as

$$Q^{-1} = n(\hbar G/c^3)^{1/2} \tag{16}$$

or more generally $\int Q\, dA = n(\hbar G/c^3)^{1/2}$, where the integral is over a surface.

3. SOME CONSEQUENCES OF A MASSIVE PHOTON

A recent paper by Vigier *et al.* (1991) discusses some cosmological consequences of a nonzero photon mass. However, the photon mass in that paper is discussed in the sense of the usual Proca theory, which is non-gauge-invariant, unlike what we have here.

In our case we should also point out the analogy with the London equation, which implies a modification of the electrodynamic equations inside a superconductor. For the London equation also we have a length parameter which gives the penetration depth which is the analog of Q^{-1} here. For our case we have, for instance, the torsion-modified equation:

$$\nabla^2 H = (\varkappa\alpha/3\pi) H \cdot Q \tag{17}$$

where the formal analogy with London's equation would again suggest Q^{-1} as the penetration depth, the photon again acquiring an effective mass, the important difference being that the London equation violates phase invariance, whereas equation (17) does not.

Since torsion is also connected with magnetism (de Sabbata and Gasperini, 1980d; de Sabbata and Sivaram, 1988, 1991), we can invoke the critical magnetic field given by

$$B_{\text{crit}} = (m_\gamma^2 c^3/q\hbar) \tag{18}$$

where q is an extra charge on the field as given by the additional term. This would also be consistent with the "superfluid" background approach of Vigier *et al.* (1991).

Using for B_{crit} the primordial field of $\approx 10^{-7}$ G and for $m_\gamma \approx 10^{-62}$ g as estimated from the background torsion, we have a constraint on the extra charge as

$$q \approx 10^{-48} e \tag{19}$$

where e is the electric charge. This is much more stringent than the limits given by the isotropy of the cosmic ray background (Sivaram, 1989), which is $q \approx 10^{-32} e$. Thus this limit on the photon mass fixed by the background torsion also constrains the photon charge to be $< 10^{-48} e$.

The well-known dispersion relation for massive photons, i.e., an effective refraction index $n \simeq 1 - (m_\gamma^2 c^4/2\hbar^2 f^2)$, would for $m_\gamma \approx 10^{-62}$ g imply $(1 - v_\gamma^2/c^2) < 10^{-26}\,\mathrm{f}^2$ and anisotropy changes of $\delta v_\gamma/c < 10^{-29}$, which are virtually unobservable.

Unlike the case of neutrinos which couple directly to torsion, photons do not couple directly to a torsion background (de Sabbata and Gasperini, 1980a). In the case of neutrinos we thus have oscillations induced by torsion even for massless neutrinos (de Sabbata and Gasperini, 1981e). We also have a magnetic moment for massless neutrinos induced by torsion (de Sabbata and Sivaram, 1990b).

It is very interesting to note that the above constraints on the torsion-induced photon mass and charge also give a photon magnetic moment of

$$\mu_\gamma = q\hbar/m_\gamma c \approx 10^{-20}\mu_\beta \tag{20}$$

(μ_β = Bohr magneton). This, although extremely small, could cause polarization changes in extremely strong magnetic fields as in neutron stars. If we have an effectively massive photon state propagating in a torsion background, we can have photon oscillations from one state of polarization to another. The oscillation distance would be given by [analogously to the case of neutrinos considered in de Sabbata and Sivaram (1990b)]

$$l_\gamma = E_\gamma/\Delta m^2_{\gamma_1} c^3 \tag{21}$$

where E_γ is the photon energy. The probability of oscillation on a propagation distance l is given by $\exp(-l_\gamma/l)$. For microwave background photons, $E_\gamma \simeq 10^{-4}$ eV. The lack of asymmetries in the microwave background would imply $\exp(-l_\gamma/l) \simeq 10^{-5}$. If we take $l_\gamma \approx R_{\mathrm{H}}$ to avoid inconsistencies, then equation (21) implies $\Delta m^2_{\gamma_1} < 10^{-52}$ g, more stringent than current experimental limits.

It would also be interesting to consider torsion effects on electrodynamics in the vicinity of a neutron star. While discussing torsion-modified massive electrodynamics some authors have introduced an arbitrary *direct* torsion–photon coupling λ which in dimensionless form can be related to Q through m_γ as (Garcia de Andrade, 1990b):

$$(m_\gamma c/\hbar)^2 \approx \lambda Q^2 \tag{22}$$

We can put a constraint on λ in considering torsion effects in neutron stars. To calculate from equation (3), we need to know the spin density. If all the neutron spins are aligned (de Sabbata and Gasperini, 1980e), i.e., $N \approx 10^{57}$ neutrons in a 1-$M_\odot$ neutron star, we have a total magnetic moment $\mu_N \times N$, where $\mu_N = (e\hbar/2m_N c)1.93 \approx e\hbar/m_N c$ is the neutron magnetic moment. Thus this would give a total magnetic moment (if all the spins are aligned) of $\simeq N\mu_N \simeq 2 \times 10^{34}$. The corresponding magnetic field is (denoting by V the volume of the neutron star) $\approx N\mu_N/V \approx 10^{15}$ G. Since the actual magnetic field of a neutron star is only $\approx 10^{12}$ G, this implies that perhaps only one in 10^3 spins is aligned, so that $N\hbar/10^3$ is the effective contribution of the total spin to the torsion–photon coupling. This can be used for the spin density to calculate Q in equation (3). Using for m_γ the value $m_\gamma \approx 10^{-62}$ g as given by the cosmological argument, we have the constraint on λ from (22) as

$$\lambda \leqslant 10^{-24} \tag{23}$$

This puts the limit on any *direct* photon–torsion coupling which *violates* gauge invariances. So this is a constraint on any *gauge violating* photon–torsion coupling.

Similar limits can be put on direct torsion couplings to massive vector mesons (such as ρ-mesons). One can also consider massive vector-photon oscillations such as the f–g oscillations (de Sabbata and Gasperini, 1986).

REFERENCES

Barnes, A., and Sergle, J. (1975). *Physical Review Letters*, **35**, 1117.
Bass, L., and Schrödinger, E. (1955). *Proceedings of the Royal Society of London A*, **232**, 1.
Chow, T. L. (1981). *Lettere al Nuovo Cimento*, **31**, 289.
De Sabbata, V., and Gasperini, M. (1980a). *Physics Letters*, **77A**, 300.
De Sabbata, V., and Gasperini, M. (1980b). *Lettere al Nuovo Cimento*, **28**, 229.
De Sabbata, V., and Gasperini, M. (1980c). *Lettere al Nuovo Cimento*, **28**, 234.
De Sabbata, V., and Gasperini, M. (1980d). *Lettere al Nuovo Cimento*, **27**, 133.
De Sabbata, V., and Gasperini, M. (1981a). *Physical Review D*, **23**, 2116.
De Sabbata, V., and Gasperini, M. (1981b). *Physics Letters*, **83A**, 115.
De Sabbata, V., and Gasperini, M. (1981c). *Lettere al Nuovo Cimento*, **30**, 363.
De Sabbata, V., and Gasperini, M. (1981d). *Lettere al Nuovo Cimento*, **30**, 503.
De Sabbata, V., and Gasperini, M. (1981e). *Nuovo Cimento A*, **66**, 479.
De Sabbata, V., and Gasperini, M. (1986). *General Relativity and Gravitation*, **18**, 669.
De Sabbata, V., and Sivaram, C. (1988). *Nuovo Cimento A*, **100**, 919.
De Sabbata, V., and Sivaram, C. (1990a). *Nuovo Cimento B*, **105**, 1181.
De Sabbata, V., and Sivaram, C. (1990b). *Nuovo Cimento B*, **105**, 603.
De Sabbata, V., and Sivaram, C. (1991). In *Modern Problems of Theoretical Physics (Festschrift for Professor D. Ivanenko)*, P. I. Pronin and Yu. N. Obukhov, eds., World Scientific, Singapore, p.143.
Einstein, A. (1917). *Annalen der Physik*, **18**, 121.
Garcia de Andrade, L. (1990a). *General Relativity and Gravitation*, **22**, 883.
Garcia de Andrade, L. (1990b). *Nuovo Cimento B*, **105**, 1297.
Hayashi, K., and Sasaki, A. (1978). *Nuovo Cimento B*, **45**, 205.
Hehl, F. W., von der Heyde, P., Kerlich, G. D., and Nester, J. M. (1976). *Review of Modern Physics*, **48**, 393.
Jackiw, R., and So-Young Pi. (1990). *Physical Review Letters*, **64**, 2969.
Novello, M. (1976). *Physics Letters*, **59A**, 105.
Sivaram, C. (1989). *Progress of Theoretical Physics*, **82**, 115.
Vigier, J.-P., *et al.* (1991). *Physics Letters*, **154A**, 203.